INTEGRATION OF BOUNDARY FINDING AND REGION-BASED SEGMENTATION USING GAME THEORY

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Abstract. Robust segmentation of structures from an image is essential for a variety of applications in biomedical image analysis. Here we propose a method that integrates region based segmentation and gradient based boundary finding using game theory in an effort to form a unified approach that is robust to noise and poor initialization. The novelty of the method is that this is a bi-directional framework whereby the two seperate modules improve their results through mutual information sharing.

Keywords: *game theory, boundary finding, region based segmentation, Maximum A posteriori probability*

1. Introduction

Precise segmentation of underlying objects in an image is very important especially for biomedical image analysis where it constitutes an important pre-processing step to such tasks as the registration of images obtained from two modalities, quantitative analysis of anatomical structures, the derivation of priors for image reconstruction in another modality and cardiac motion tracking. Two basic approaches to image segmentation are as follows. On one hand, we have region-based methods, which rely on the homogeneity of spatially localized features and other pixel statistics. The other group of work consists of gradient-based edge detection and boundary finding methods. However, the success of either of the methods is often limited by poor image quality. Integration of these methods can potentially result in an improved output as pointed out in [9]. However, integration always result in a larger complicated system, analyzing which is a daunting task. Probably the easiest way to do that is to divide the system into functional modules and analyze them separately. Hence, the reverse problem i.e. the development of such large and complex systems (which is our goal) often consists of intelligently assembling smaller well-understood modules, leading to what can correctly be termed *integrated* systems. Here *integration* refers not only to succinctly describing the mathematical relations between these modules but also describing algorithms that can perform the integration.

Thus in effect, we have in the general case, a team of modules (two for the present segmentation problem) working in a coordinated fashion in a possibly hostile and uncertain environment towards common or at least related goals. It is important to note that the members do not necessarily acquire the same information and thus may need to operate in a decentralized mode of decision making. If we presume the presence of a common probability space, thereby allowing the modules to share a similar view of the world, this could lead to probably the most common method of optimizing multi-objective functions, namely the single objective function approach.

Since the single objective approach seems to often be the method of choice in the vision literature [7, 1], we note that there are occasions when it is inadequate or at least not the best approach. One of the important concerns could be the robustness of such models and the *optimum* solutions that it produces under small variations in the underlying assumptions. This could especially be important in situations where the models perceive the outside world differently i.e. if they have different probability descriptions of the environment. This would then lead to different expected cost functions for the different modules, making the use of a single objective approach at best sub-optimal. One of the biggest advantages of using game theory is that it can dealing with this problem by treating it as a stochastic non-zero sum game with the modules taking on the roles of decision makers having different subjective probability measures.

Another advantage of using the game-theoretic approach is that the modules need not know the cost functions of the others. This keeps them decoupled and hence, computationally light. The flow of information is restricted to passing only the results of the decisions between the modules. Thus we could combine for example incommensurate objectives through this approach which might not be possible using the conventional single objective approach.

Coming back to the integration problem, both region based-methods, and gradient-based methods have their advantages and their disadvantages. While the region-based methods can handle better noise and loss of high frequency information, gradient-based methods are less error prone to changes in the grey scale values and can deal in a better way with shape changes. Our aim in this paper is to combine the strengths of both the approaches in an integrated method.

Among the available methods, AI based techniques have been used where production rules are invoked for conflict removal [9]. Other efforts have used probability-based approaches (see e.g. [5, 1]) where often the aim is to maximize the *a posteriori* probability of the region classified image given the raw image data by optimization methods like simulated annealing. Integration here is achieved in the local or dense field sense where the edges are used as line processes and the optimization is achieved both over the location of the line processes as well as the pixel classification.

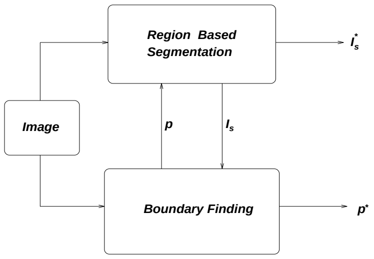

Figure 1. Flow diagram for game-theoretic integration of region based segmentation and boundary finding. The outputs of each of the models, I_s for the region module and p for the boundary module are fed back to each other after every decision making step. The game stops when none of the modules can improve their positions unilaterally. The final solution pair, (I_s^*, p^*) constitutes the Nash Equilibrium

The main contribution of this paper is thus to present an integrated bi-directional approach for segmentation by combining boundary finding and region based segmentation using a game-theoretic framework. This leads to a system where the two modules would operate in parallel so that at each step the outputs of each model get updated using information from the outputs of both the models from the previous iteration. Thus as the game progresses, both the modules improve their positions through mutual information sharing. The final boundary obtained is a compromise between the region segmented image and the gradient image, the edges are better localized. On the other hand, the region based segmentation is forced to be consistent with the presence of real objects in the image, making small islands of mis-segmented pixels highly unlikely. Further, we also present a theorem that concerns with the existence of the Nash Equilibrium. It is related to a theorem in [6, 2] but is more relevant to the present segmentation problem.

2. Game Theory

In this section we briefly introduce the concepts of parallel decision making and state a theorem for the stability and existence of a Nash Equilibrium for the types of objective functions that we would be using for our segmentation problem. The game is being played by a set of decision makers (or players) which for our case will correspond to the two segmentation modules that we try to integrate. Let $N = 1, 2$ be the player set, P^1 and P^2 be the strategy spaces of the decision makers D1 and D2 respectively (i.e. $p^1 \in P^1$ and $p^2 \in P^2$ constitute their strategies) and let

$$F^1 \ : P^1 \times P^2 \longrightarrow R \ ; \ F^2 \ : P^1 \times P^2 \longrightarrow R$$

be the cost function for the decision makers 1 and 2 respectively. Then the normal form of the game is described by $\{F^1, F^2\}$ where it is assumed that P^1 and P^2, the strategy spaces are also Hilbert spaces and the decision makers strive to minimize their cost functions at each turn. This constitutes a rational form of decision making for the individual players.

The game continues unless the decision makers cannot unilaterally improve their positions (i.e. further minimize their cost functions). This natural stopping point is termed as the Nash Equilibrium. Formally, a pair of strategies ($\bar{p}^1 \in P^1$, $\bar{p}^2 \in P^2$) constitutes a Nash Equilibrium solution if

$$\forall p^1, p^2 \quad F^1(\bar{p}^1, \bar{p}^2) \leq F^1(p^1, \bar{p}^2) \ ; \ F^2(\bar{p}^1, \bar{p}^2) \leq F^2(\bar{p}^1, p^2) \qquad (1)$$

The Nash Equilibrium is obtained by a set of rational choices made by the players. If we consider k to be the time index, then this procedure of parallel decision making can be viewed as

$$p^1_{k+1} = \arg \min_{p^1 \in P^1} F^1(p^1, p^2_k) \ ; \ p^2_{k+1} = \arg \min_{p^2 \in P^2} F^2(p^1_k, p^2) \qquad (2)$$

where $p^1_0 \in P^1$ and $p^2_0 \in P^2$ is the initial point. A Nash Equilibrium is stable if it can be obtained as the limit of the above for any initial point. It is locally stable if convergence is valid for all initial conditions in some ϵ-neighborhood of the equilibrium solution.

For our segmentation problem, we shall be concerned with cost functions of the following structure

$$F^1(p^1, p^2) = f_1(p^1) + \alpha f_{21}(p^1, p^2) \ ; \ F^2(p^1, p^2) = f_2(p^2) + \beta f_{12}(p^1, p^2) \qquad (3)$$

where α and β are some scaling constants.
We also consider the following constraints on F^1 and F^2.
(i) F^i is bounded in $p^i \in P^i$.
(ii) F^i is continuously second order differentiable in $p^i \in P^i$.
(iii) $\exists$ a closed neighborhood $u^i \subseteq P^i$ such that F^i is strongly convex in u^i.
For the above assumptions, we have the following:
Theorem: For the given above structure of $F^1(p^1, p^2)$ and $F^2(p^1, p^2)$ there exists a locally stable Nash Equilibrium solution, ie. for any $p^1 \in U^1 \subseteq P^1$ and $p^2 \in U^2 \subseteq P^2$, the sequence of rational choices generated by the parallel

decision making process converges and the limit point is a Nash equilibrium solution if α and β satisfy the following condition:

$$\left\| \left[\left(\alpha^{-1} \frac{\partial^2}{\partial p^1 \partial p^1} f_1(p^1) + \frac{\partial^2}{\partial p^1 \partial p^1} f_{21}(p^1, p^2) \right)^{-1} \left(\frac{\partial^2}{\partial p^1 \partial p^2} f_{21}(p^1, p^2) \right) \right] \right.$$
$$\left. * \left[\left(\beta^{-1} \frac{\partial^2}{\partial p^2 \partial p^2} f_2(p^2) + \frac{\partial^2}{\partial p^2 \partial p^2} f_{12}(p^1, p^2) \right)^{-1} \left(\frac{\partial^2}{\partial p^2 \partial p^1} f_{12}(p^1, p^2) \right) \right] \right\| < 1$$

What this theorem tells us is that there exists a Nash Equilibrium and also it states a condition under which that exists provided the modules described by their corresponding objective functions are well behaved. But, assuring the above condition is non trivial. However, as can easily be verified, for simple objectives this does not severely constrain the problem. For complex systems as in the present case, it is almost impossible to analytically verify the above, but most reasonable choices of α and β results in convergence.

3. The modules

The modules that we consider here are the two types of segmentation procedure (see Figure 1), one region based and the other gradient based boundary finding. Even though the method works for other more general image models, for simplicity of analysis, we shall assume that the image can be modeled by

$$y(i,j) = x(i,j) + n(i,j); \quad 1 \le i \le M; \quad 1 \le j \le N \tag{4}$$

where $n(i,j)$ corresponds to additive white Gaussian noise (AWGN). Now, as for $x(i,j)$, we make the further assumption that it can be modeled as a collection of homogeneous regions of uniform or slowly varying intensities.

3.1. REGION BASED SEGMENTATION MODULE

Here we try to describe the region based segmentation model. For this one we have $Y = I$ corresponds to the image data and X corresponds to the segmented image consisting of uniform regions. The task of this module is to come up with a segmentation X given the raw image Y and our prior knowledge of the process X. Thus the aim is $\arg\max_X P(X|Y)$. We model the region process X by a Markov random field. For recovering piecewise smooth functions as hypothesized in this problem, we shall use the weak membrane model [1]. Hence, maximizing the *a posteriori* probability in the above equation becomes equivalent to minimizing the energy function

$$E = \sum_{i,j} [y_{i,j} - x_{i,j}]^2 + \lambda^2 \left(\sum_{i,j} (x_{i,j} - x_{i-1,j})^2 + \sum_{i,j} (x_{i,j} - x_{i,j+1})^2 \right) + P \tag{5}$$

In the above equation, the first term is the data fidelity term, the second enforces smoothness and the last term is the penalty term which is non-zero if we want to recover the line processes. But, again for the simplicity of analysis we shall ignore that term and instead assume that the image intensities change smoothly over the regions. Such a model works reasonably for most of the situations. However, in some cases it becomes necessary to incorporate the line processes.

3.2. BOUNDARY FINDING MODULE

In this module, the aim is to find a class of objects with smooth boundaries that are continuously deformable. A variety of deformable contour methods have been proposed. We use the approach due to Staib and Duncan [10] which uses a Fourier parameterization which we have adopted in this work as our primary approach. It expresses a curve in terms of an orthonormal basis. Thus

$$v(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{bmatrix} a_0 \\ c_0 \end{bmatrix} + \sum_{k=1}^{\infty} \begin{bmatrix} a_k & b_k \\ c_k & d_k \end{bmatrix} \begin{bmatrix} cos(kt) \\ sin(kt) \end{bmatrix} \tag{6}$$

where, $v(t)$ is the contour vector consisting of the x and y coordinates and $p = (a_0, c_0, a_1, b_1, c_1, d_1, ...)$ represent the parameterized boundary. The estimation of these parameters to find the boundary is posed as an optimization problem, where the objective function measures the strength of the boundary given the set of parameters from the image. (see [10] for more details)

4. Interacting Modules

Here, in this section, we shall describe the interaction between the two modules. This would represent a modification of the cost function and we shall show that the format of the terms will represent that in the parallel decision theoretic model that we had described earlier.

4.1. REGION BASED SEGMENTATION INFLUENCED BY BOUNDARY FINDING

We have already described the region based method. Initially the requirement was to develop a region model given the raw image and the prior knowledge which was modeled using a MRF which in fact was no more than a smoothness constraint resulting in what is termed as a weak membrane model. Now, besides the input image and the smoothness constraint, we also have as a prior the output of the boundary finder in terms of a contour p. Thus the aim is $\arg\max_X P(X|Y, p)$, where as already defined, Y is the raw image data and X is the segmented image. Since we are concerned with maximization w.r.t. X, our aim is

$$\arg\max_X lnP(X|Y,p) \equiv \arg\max_X lnP(X|Y) \, + \, \arg\max_X lnP(p|X) \qquad (7)$$

The first term in the above equation is derived from the image data and the previous output of this module, whereas the second term represents the influence of the previous output of the boundary finding module.

As we can recognize, the first term in the above equation (7) is the same as that in the original stand alone model. The second term, however corresponds to the information derived from the boundary finding module.

Even though this procedure can be extended to images with more than two regions, for the time being we shall continue with the assumption that there are only two homogeneous regions, one of which represents the target object. Thus the contour vector p represents the boundary of that object. Now, let A_p correspond to the points that lie inside the contour and $\bar{A}_p$ those points that lie on the background. Thus, $A_p \bigcup \bar{A}_p = \{(i,j);\ 1 \le i \le M,\ 1 \le j \le N\}$ i.e. the whole image consists of those two parts only.

A very simple model to represent this information is thus given by,

$$lnP(p|X) \propto \sum_{(i,j)\in A_p} (x_{i,j} - u_{i,j})^2 + \sum_{(i,j)\in \bar{A}_p} (x_{i,j} - v_{i,j})^2 \qquad (8)$$

where $u_{i,j}$ represents the information regarding the intensity of those points that lies inside the contour and $v_{i,j}$ of those that lie outside. For most simple images with uniform regions, they could actually represent just the expected value of the pixels at those points, something that can easily be computed *a priori* [8]. Thus, finally the region based module including the interacting term is mathematically equivalent to the minimization of the following:

$$\begin{aligned}
\min_X E &= \min_X \Bigg[\sum_{i,j} \Big([y_{i,j} - x_{i,j}]^2 + \lambda^2 [(x_{i,j} - x_{i-1,j})^2 + (x_{i,j} - x_{i,j+1})^2] \Big) \\
&+ P + \alpha \left\{ \sum_{(i,j)\in A_p} (x_{i,j} - u_{i,j})^2 + \sum_{(i,j)\in \bar{A}_p} (x_{i,j} - v_{i,j})^2 \right\} \Bigg] \qquad (9) \\
&= \min_X [f_1(X) + \alpha f_{21}(X,p)]
\end{aligned}$$

The thing to note here is the fact that we were able to pose the problem in the format of the parallel decision making module as shown in the above, with the first term of the objective function $f_1(X)$ representing the contribution of the region based module by itself and the other term $f_{21}(X,p)$ representing the interaction term which uses the latest available output p of the boundary module.

4.2. BOUNDARY FINDING INFLUENCED BY REGION BASED SEGMENTATION

Unlike the stand alone boundary finding model, here, besides the gradient image, we will also use as input, the output from the region based method. Once again, the entire problem will be posed in a Bayesian framework as we shall shortly see. Thus, we want to maximize $P(p|I_g, I_s)$, where as described in the previous section, p is the vector of parameters used to parameterize the contour and I_g represents the gradient image and I_s (obtained from X of the region module) is the region segmented image. It can be shown that our aim is [4]

$$\begin{aligned} \max_p P(p|I_g, I_s) &\propto \max_p P(I_g|p)P(I_s|p)P(p) & (10) \\ &\equiv \max_p [lnP(p) + lnP(I_g|p) + lnP(I_s|p)] & (11) \end{aligned}$$

The first two terms in (11) as we can recognize, are identical to the terms that were present in the stand alone boundary finder and have the same significance as before.

The last term, however incorporates the region based information into the boundary finding framework. It can be shown, (see [4] for details) that this term can be represented as

$$lnP(I_s|p) \propto \int\int_{A_p} I_s(x, y)dA \qquad (12)$$

The intuition, here is that if by any means we could identify the target region by positive values, and the other by negative values, the above area integral is maximum when the boundary correctly encircles the target region. For the simple two region problem, after the segmentation has been done, we could linearly translate the intensity values to satisfy the above requirement. Here, that would just mean subtracting out the mean of the means of the two characteristic regions. In other words for this case, $I_s(i, j) = X(i, j) - m$, where (i, j) represents the pixel location and m is the mean of the means of the two regions.

Omitting the details, it can be derived that we finally have:

$$\begin{aligned} &\max_p P(I_g|I_s, p)P(I_s|p)P(p) \\ &= \max_p \left(\left[lnP(p) + \int_{C_p} K_1 I_g(x(p,t), y(p,t))dt \right] \right. \\ &\left. + \beta \left[\int_{C_p} \{N_s(x(p,t), y(p,t)) \frac{\partial x}{\partial t} + M_s(x(p,t), y(p,t)) \frac{\partial y}{\partial t}\} \, dt \right] \right) \quad (13) \\ &= \max_p [f_2(p) + \beta f_{12}(I_s, p)] \end{aligned}$$

Here, K_1 is a constant and so is β. However, the point to note is that the construct of the objective function is once again consistent with the game theoretic framework where $f_2(p)$ is the contribution of the gradient based boundary finder and $f_{12}(I_s, p)$ is the interaction term that uses the latest available output from the region module.

Thus, in this section we have presented the segmentation problem in a game-theoretic framework where the players correspond to the two different segmentation procedures. At each turn, each module tries to find a new output that best satisfies it's cost function based on the previous set of outputs. It then updates it's former output to this new output and broadcasts it to the other module as in Figure 1. The game finishes when both the modules have converged to their optimal strategies (see [3] for further details). That this process of decision making is acceptable and that the algorithm indeed converges to the Nash equilibrium has been shown in [6, 2].

5. Results

Experiments were carried out both with synthetic and real biomedical images to verify the performance of game theoretic integration against the individual models without any information sharing. The equations (9) and (13) jointly describe the outputs of the game-theoretic integration where equation (9) gives the region output and equation (13) the boundary output under the integrated framework. When the coupling coefficients are set to zero, we have the stand alone models.

First, we use a synthetic image that has one target object in the center surrounded by a background as shown in Figure 2 where the SNR is 0.67. The outputs of the two modules with and without game theoretic integration are shown in the above figure. Clearly, both the modules improve their outputs under game theoretic integration.

In the next figure we try to analyze the effects of integration for both the modules under increasing amounts of noise. In Figure 3 (a) we compare the output of stand alone region based segmentation with the same output obtained using game-theoretic integration as in equation (9). In Figure 3 (b) we compare the output of stand alone gradient-based boundary finder with the same output obtained using game-theoretic integration as in equation (13). We use the same template as in the last figure. The contrast difference between the background and the target is 200.0 and the x-axis in the Figures 3 (a) and 3 (b) correspond to the standard deviation σ for the white noise added. Both the experiments vary from $\sigma = 25$ to $\sigma = 400$. In other words, the SNR varies from 8.0 to 0.5. As for the y-axis, in Figure 3 (a), it represents the average per-pixel intensity error, and in Figure 3 (b), it represents a scaled version of the average Euclidean distance between

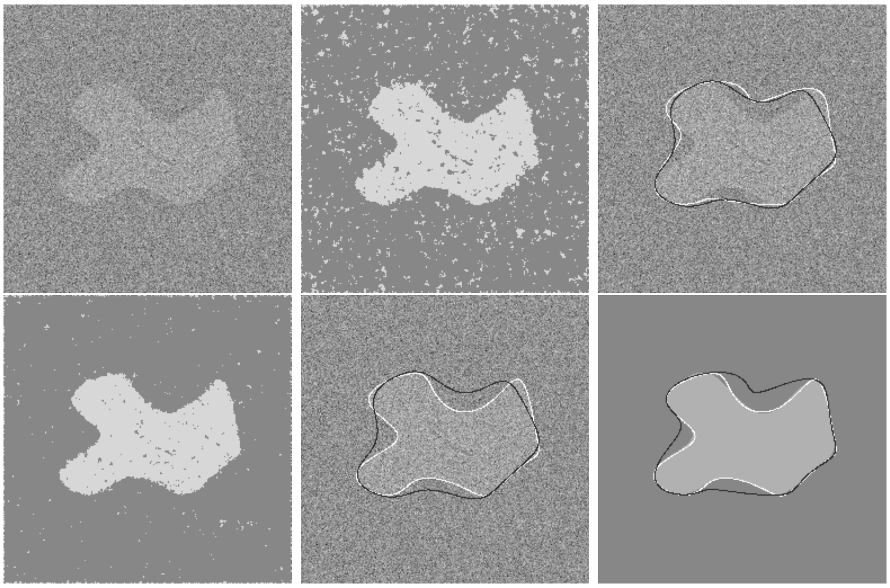

Figure 2. Example with a synthetic image to show the effects of game-theoretic integration. (a)Top,Left: Original image with $SNR = 0.67$. (b)Top,Middle: Output of region based segmentation without using game-theoretic integration. (c)Top,Right: Output of boundary finding without using game-theory; The dark contour represents the initialization, and the brighter one is the final output. (d)Bottom,Left: Output of region based segmentation using game-theoretic integration. (e)Bottom,Middle: Output of boundary finding using game-theory; The dark contour represents the initialization, and the brighter one is the final output. (f)Bottom,Right: The final boundary outputs generated by the two methods overlayed on top of the template. The dark contour represents the output of the gradient-based boundary-finder without using game-theory, and the brighter one is the final output using game-theoretic integration.

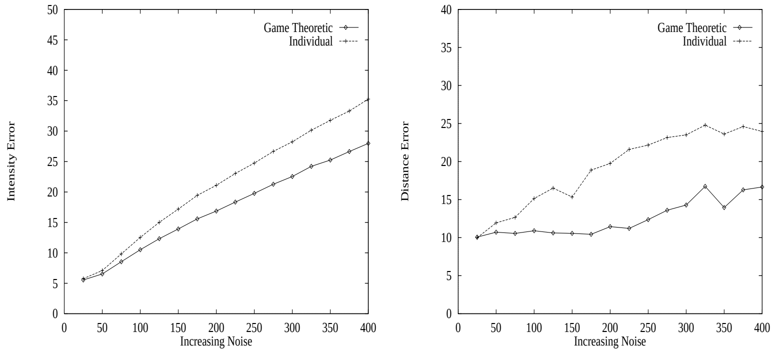

Figure 3. Performance of the two modules under increasing noise conditions without and with using game theoretic integration (a)Left: Comparison of region module with and without game-theoretic information sharing. (b)Right: Comparison of boundary finding with and without game-theoretic information sharing. Clearly, the error is larger without information sharing that what we get using game-theoretic integration for both the cases.

the true contour and the final one obtained (see [4] for details). Clearly, the results are superior for both the modules when there is game-theoretic information fusion.

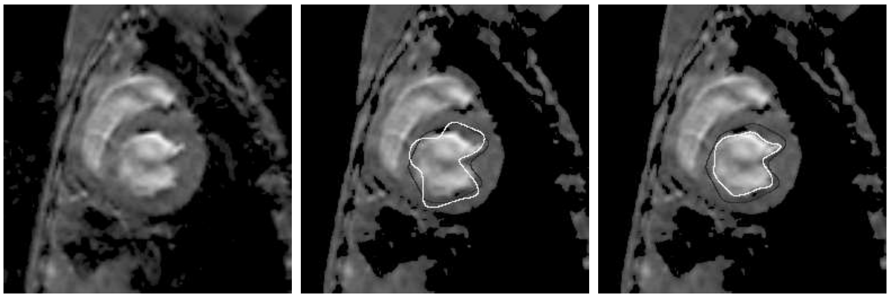

Figure 4. Example with the short axis MR image of a canine heart. (a)Left: Original image; (b)Middle: Output of gradient based boundary finding without using game theoretic integration. The dark contour represents the initialization, and the brighter one is the final output. (b)Right: Output of gradient based boundary finding using game theoretic integration. The dark contour represents the initialization, and the brighter one is the final output.

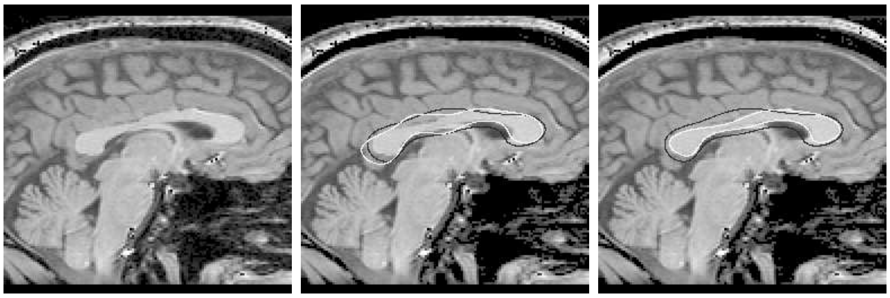

Figure 5. Example with the MR image of the brain. (a)Left: Original image; (b)Middle: Output of gradient based boundary finding without using game theoretic integration. The dark contour represents the initialization, and the brighter one is the final output. (b)Right: Output of gradient based boundary finding using game theoretic integration. The dark contour represents the initialization, and the brighter one is the final output.

In Figures 4 and 5 we show the results of game theoretic integration for some typical clinical images. While in the heart image, noisy and fuzzy edges seem to be the problem, in the brain image the problem seems to be the poor initialization. However, the game theoretic outputs in both the cases are satisfactory. This clearly demonstrates the robustness of the game theoretic method to both noise and poor initialization.

6. Conclusions

We have presented in this paper a new technique for integrating region based segmentation and gradient based boundary finding using the game theoretic framework. We have posed this as a non-zero sum two-person non-cooperative game where, the cost functions of both the modules are based on the Bayesian theory of maximization of the *a posteriori* probability. The main advantage of using the game-theoretic integration is that it can bring together the incommensurate region and boundary methods that operate in different probability spaces into a common information sharing framework of rational decision making. This leads to an improved output for both the modules. As the examples show, the integrated approach is much more robust to both noise as well as poor initialization. Application of this method on real medical images results in noticeable improvement as has been shown.

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