Homework 1 (ENAS913)

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[**1**] This question is about solving word problems by translating them into set theoretic problems: Is the oldest engineer who plays chess the same as the oldest chess player who is an engineer?

[**2**] Prove deMorgan's Theorem: $\widetilde{A \cap B} = \tilde{A} \cup \tilde{B}$.

[**3**] Sometimes it is necessary to consider operators that produce more than one output for a given input. For example if $x \geq 0$, then the square root operator may be defined as producing two outputs $\sqrt{x}$ and $-\sqrt{x}$. Write this square root operator as a function. Be careful about defining the domain and co-domain of your function.

[**4**] Let A and B be two events such that $A \subset B$. If $P(A) = 1/4$ and $P(B) = 1/3$, calculate $P(A \mid B)$ and $P(B \mid A)$.

[**5**] A friend suggests to you that if x is any random variable then its cdf F_x and pdf f_x are always related by first-order differentiation according to

$$f_x = \|\nabla F_x\|.$$

Find a counter-example to prove your friend wrong.

[**6**] Let u be a scalar-valued ($\mathcal{R}^1$-valued) random variable and $v = h(u)$, where

$$h(x) = \begin{cases} 0 & \text{if } x \leq 0 \\ \frac{1}{x} & \text{if } x > 0. \end{cases}$$

Calculate the cdf and pdf of v in terms of the cdf and pdf of u.

[**7**] Let x, y be random variables and let $z = xy$. Calculate the pdf of z in terms of the joint pdf of x, y using recipe 4 in the notes.